The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING

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Topic Generator - Solution Set
Solutions

1. $4.1 + 1.05 + 2.005$ equals
(A) 7.155 (B) 7.2 (C) 8.1 (D) 7.605 (E) 8.63

Source: 2009 Gauss Grade 7 #1

Primary Topics: Number Sense

Secondary Topics: Operations | Decimals

Answer: A

Solution:

Adding, $4.1 + 1.05 + 2.005 = 5.15 + 2.005 = 7.155$.

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2. Rounded to 2 decimal places, $\frac{7}{9}$ is
(A) 0.7 (B) 0.77 (C) 0.78 (D) 0.79 (E) 0.8

Source: 2009 Gauss Grade 8 #5

Primary Topics: Number Sense

Secondary Topics: Decimals | Fractions/Ratios

Answer: C

Solution:

Calculating, $\frac{7}{9} = 7 \div 9 = 0.7777\cdots = 0.\overline{7}$. Rounded to 2 decimal places, $\frac{7}{9}$ is 0.78.

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3. $\frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3}$ equals
(A) $3\frac{1}{3}$ (B) $7 + \frac{1}{3}$ (C) $\frac{3}{7}$ (D) $7 + 3$ (E) $7 \times \frac{1}{3}$

Source: 2011 Gauss Grade 7 #7

Primary Topics: Number Sense

Secondary Topics: Fractions/Ratios

Answer: E

Solution:

Since we are adding $\frac{1}{3}$ seven times, then the result is equal to $7 \times \frac{1}{3}$.

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4. The largest fraction in the set $\left\{\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{10}\right\}$ is
(A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{1}{4}$ (D) $\frac{1}{5}$ (E) $\frac{1}{10}$

Source: 2012 Gauss Grade 7 #4

Primary Topics: Number Sense

Secondary Topics: Fractions/Ratios

Answer: A

Solution:

A positive fraction increases in value as its numerator increases and also increases in value as its denominator decreases.
Since the numerators of all five fractions are equal, then the largest of these is the fraction with the smallest denominator.
The largest fraction in the set $\left\{\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{10}\right\}$ is $\frac{1}{2}$.

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5. Which of the following is *not equal* to a whole number?
(A) $\frac{60}{12}$ (B) $\frac{60}{8}$ (C) $\frac{60}{5}$ (D) $\frac{60}{4}$ (E) $\frac{60}{3}$

Source: 2012 Pascal Grade 9 #3

Primary Topics: Number Sense

Secondary Topics: Fractions/Ratios

Answer: B

Solution:

Since $\frac{60}{8} = 60 \div 8 = 7.5$, then this choice is not equal to a whole number.
Note as well that $\frac{60}{12} = 5$, $\frac{60}{5} = 12$, $\frac{60}{4} = 15$, and $\frac{60}{3} = 20$ are all whole numbers.

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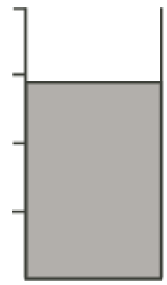
6. A large cylinder can hold 50 L of chocolate milk when full. The tick marks show the division of the cylinder into four parts of equal volume. Which of the following is the best estimate for the volume of chocolate milk in the cylinder as shown?



(A) 24 L (B) 28 L (C) 30 L (D) 36 L (E) 40 L

Source: 2013 Cayley Grade 10 #6

Primary Topics: Geometry and Measurement

Secondary Topics: Estimation | Volume | Measurement | Fractions/Ratios

Answer: D

Solution:

Since the tick marks divide the cylinder into four parts of equal volume, then the level of the milk shown is a bit less than $\frac{3}{4}$ of the total volume of the cylinder.
Three-quarters of the total volume of the cylinder is $\frac{3}{4} \times 50 = 37.5$ L.
Of the five given choices, the one that is slightly less than 37.5 L is 36 L, or (D).

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7. 30% of 200 equals

(A) 0.06 (B) 0.6 (C) 6 (D) 60 (E) 600

Source: 2014 Pascal Grade 9 #3

Primary Topics: Number Sense

Secondary Topics: Percentages

Answer: D

Solution:

30% of 200 equals $\frac{30}{100} \times 200 = 60$.
Alternatively, we could note that 30% of 100 is 30 and $200 = 2 \times 100$, so 30% of 200 is 30×2 which equals 60.

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8. Three thousandths is equal to
   (A) 300 (B) 0.3 (C) 0.03 (D) 30 (E) 0.003

Source: 2017 Gauss Grade 7 #5

Primary Topics: Number Sense

Secondary Topics: Operations | Decimals

Answer: E

Solution:

Written as a fraction, three thousandths is equal to $\frac{3}{1000}$.
As a decimal, three thousandths is equal to $3 \div 1000 = 0.003$.

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9. The expression $\frac{3}{10} + \frac{3}{100} + \frac{3}{1000}$ is equal to
   (A) 0.333 (B) 0.9 (C) 0.963 (D) 0.369 (E) 0.30303

Source: 2023 Pascal Grade 9 #5

Primary Topics: Number Sense | Algebra and Equations

Secondary Topics: Fractions/Ratios | Expressions | Decimals

Answer: A

Solution:

Evaluating, $\frac{3}{10} + \frac{3}{100} + \frac{3}{1000} = 0.3 + 0.03 + 0.003 = 0.333$.

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10. A package of 8 greeting cards comes with 10 envelopes. Kirra has 7 cards but no envelopes. What is the smallest number of packages that Kirra needs to buy to have more envelopes than cards?
    (A) 3 (B) 4 (C) 5 (D) 6 (E) 7

Source: 2022 Gauss Grade 8 #9

Primary Topics: Counting and Probability | Number Sense

Secondary Topics: Counting | Equations Solving | Expressions | Fractions/Ratios

Answer: B

Solution:

Each package of greeting cards comes with $10 - 8 = 2$ more envelopes than cards.
Thus, 3 packages of greeting cards comes with $3 \times 2 = 6$ more envelopes than cards, and 4 packages of greeting cards comes with $4 \times 2 = 8$ more envelopes than cards.
Kirra began with 7 cards and no envelopes.
To have more envelopes than cards, Kirra must buy enough packages to make up the difference between the number of cards and the number of envelopes (which is 7).
Thus, the smallest number of packages that Kirra must buy is 4.
Note: We can check that if Kirra buys 3 packages she has $3 \times 8 + 7 = 31$ cards and $3 \times 10 = 30$ envelopes, and thus fewer envelopes than cards. However, if she buys 4 packages, she has $4 \times 8 + 7 = 39$ cards and $4 \times 10 = 40$ envelopes, and thus more envelopes than cards, as required.

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11. When the numbers $5.07\overline{6}$, $5.0\overline{76}$, 5.07, 5.076, $5.\overline{076}$ are arranged in increasing order, the number in the middle is
(A) $5.07\overline{6}$ (B) $5.0\overline{76}$ (C) 5.07 (D) 5.076 (E) $5.\overline{076}$

Source: 2009 Pascal Grade 9 #11

Primary Topics: Number Sense

Secondary Topics: Decimals

Answer: E

Solution:

We write out the five numbers to 5 decimal places each, without doing any rounding:

$$5.07\overline{6} = 5.07666\ldots 5.0\overline{76} = 5.07676\ldots 5.07 = 5.070005.076 = 5.076005.\overline{076} = 5.07607\ldots$$

We can use these representations to order the numbers as

$$5.07000, 5.07600, 5.07607\ldots, 5.07666\ldots, 5.07676\ldots$$

so the number in the middle is $5.\overline{076}$.

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12. The distance from Coe Hill to Calabogie is 150 kilometres. Pat leaves Coe Hill at 1:00 p.m. and drives at a speed of 80 km/h for the first 60 km. How fast must he travel for the remainder of the trip to reach Calabogie at 3:00 p.m.?
(A) 65 km/h (B) 70 km/h (C) 72 km/h (D) 75 km/h (E) 90 km/h

Source: 2009 Pascal Grade 9 #19

Primary Topics: Number Sense

Secondary Topics: Rates

Answer: C

Solution:

Since Pat drives 60 km at 80 km/h, this takes him $\frac{60 \text{ km}}{80 \text{ km/h}} = \frac{3}{4}$ h.
Since Pat has 2 hours in total to complete the trip, then he has $2 - \frac{3}{4} = \frac{5}{4}$ hours left to complete the remaining $150 - 60 = 90$ km.
Therefore, he must travel at $\frac{90 \text{ km}}{\frac{5}{4} \text{ h}} = \frac{360}{5} \text{ km/h} = 72 \text{ km/h}$.

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13. A survey of 400 students at Cayley University found that the ratio of students who commute to students who live on campus is $3:2$. A survey of 600 students at Fermat University found that the ratio of students who commute to students who live on campus is $2:3$. When considering all the surveyed students from both universities, what is the ratio of students who commute to students who live on campus?

(A) $2:3$ (B) $12:13$ (C) $1:1$ (D) $6:5$ (E) $3:2$

Source: 2010 Pascal Grade 9 #13

Primary Topics: Number Sense

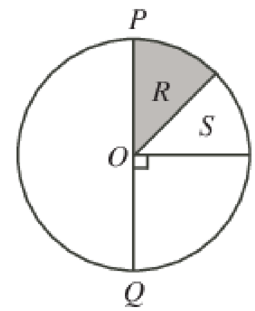
Secondary Topics: Fractions/Ratios

Answer: B

Solution:

Since the ratio of commuters to students who live on campus at Cayley University~is $3:2$, then $\frac{3}{3+2}=\frac{3}{5}$ of the students at Cayley University are commuters.
Thus, there are $\frac{3}{5}(400)=\frac{1200}{5}=240$ commuters at Cayley University
Since the ratio of commuters to students who live on campus at Fermat University~is $2:3$, then $\frac{2}{2+3}=\frac{2}{5}$ of the students at Fermat University~are commuters.
Thus, there are $\frac{2}{5}(600)=\frac{1200}{5}=240$ commuters at Fermat University.
There are $400+600=1000$ students in total at the two schools.
Of these, $240+240=480$ are commuters, and so the remaining $1000-480=520$ students live on campus.
Therefore, the overall ratio of commuters to students who live on campus is
$480:520=48:52=12:13$.

14. On the spinner shown, PQ passes through centre O. If areas labelled R and S are equal, then what percentage of the time will a spin stop on the shaded region?



(A) 50% (B) 22.5% (C) 25% (D) 45% (E) 12.5%

Source: 2012 Gauss Grade 7 #13

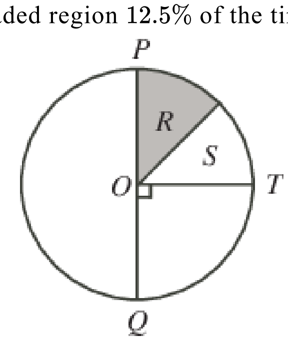
Primary Topics: Geometry and Measurement

Secondary Topics: Probability | Percentages

Answer: E

Solution:

Since PQ passes through centre O, then it is a diameter of the circle.
Since $\angle QOT = 90^\circ$, then $\angle POT = 180^\circ - 90^\circ = 90^\circ$.
Thus, the area of sector POT is $\frac{90^\circ}{360^\circ} = \frac{1}{4}$ or 25% of the area of the circle.
Since the areas labelled R and S are equal, then each is
$25\% \div 2 = 12.5\%$ of the area of the circle.
Therefore, a spin will stop on the shaded region 12.5% of the time.



15. The ratio of junior kindergarteners to senior kindergarteners at Gauss Public School is 8 : 5. If there are 128 junior kindergarteners at the school, then how many kindergarteners are there at the school?
(A) 218 (B) 253 (C) 208 (D) 133 (E) 198

Source: 2012 Gauss Grade 7 #17

Primary Topics: Number Sense

Secondary Topics: Fractions/Ratios

Answer: C

Solution:

Solution 1
Since the ratio of JKs to SKs is 8 : 5, then for every 5 SKs there are 8 JKs.
That is, the number of SKs at Gauss Public School is $\frac{5}{8}$ of the number of JKs.
Since the number of JKs at the school is 128, the number of SKs is $\frac{5}{8} \times 128 = \frac{640}{8} = 80$.
The number of kindergarteners at the school is the number of JKs added to the number of SKs or $128 + 80 = 208$.
Solution 2
Since the ratio of JKs to SKs is 8 : 5, then for every 8 JKs there are $8 + 5 = 13$ students.
That is, the number of kindergarteners at Gauss Public School is $\frac{13}{8}$ of the number of JKs.
Since the number of JKs at the school is 128, the number of kindergarteners is
$\frac{13}{8} \times 128 = \frac{1664}{8} = 208$.

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16. A bicycle at Store P costs $200. The regular price of the same bicycle at Store Q is 15% more than it is at Store P. The bicycle is on sale at Store Q for 10% off of the regular price. What is the sale price of the bicycle at Store Q?
(A) $230.00 (B) $201.50 (C) $199.00 (D) $207.00 (E) $210.00

Source: 2014 Gauss Grade 8 #19

Primary Topics: Number Sense

Secondary Topics: Percentages

Answer: D

Solution:

At Store Q, the bicycle's regular price is 15% more than the price at Store P, or 15% more than $200.
Since 15% of 200 is $\frac{15}{100} \times 200 = 0.15 \times 200 = 30$, then 15% more than $200 is $\$200 + \$30$ or~$230.
This bicycle is on sale at Store Q for 10\% off of the regular price, $230.
Since 10% of 230 is $\frac{10}{100} \times 230 = 0.10 \times 230 = 23$, then 10% off of $230 is $\$230 - \$23$ or $207.
The sale price of the bicycle at Store Q is $207.

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17. Chris received a mark of 50% on a recent test. Chris answered 13 of the first 20 questions correctly. Chris also answered 25% of the remaining questions on the test correctly. If each question on the test was worth one mark, how many questions in total were on the test?

(A) 23 (B) 38 (C) 32 (D) 24 (E) 40

Source: 2016 Pascal Grade 9 #19

Primary Topics: Number Sense

Secondary Topics: Percentages

Answer: C

Solution:

Suppose that there were n questions on the test.
Since Chris received a mark of 50% on the test, then he answered $\frac{1}{2}n$ of the questions correctly.
We know that Chris answered 13 of the first 20 questions correctly and then 25% of the remaining questions.
Since the test has n questions, then after the first 20 questions, there are $n-20$ questions.
Since Chris answered 25% of these $n-20$ questions correctly, then Chris answered $\frac{1}{4}(n-20)$ of these questions correctly.
The total number of questions that Chris answered correctly can be expressed as $\frac{1}{2}n$ and also as $13+\frac{1}{4}(n-20)$.
Therefore, $\frac{1}{2}n = 13+\frac{1}{4}(n-20)$ and so $2n = 52+(n-20)$, which gives $n=32$.
(We can check that if $n=32$, then Chris answers 13 of the first 20 and 3 of the remaining 12 questions correctly, for a total of 16 correct out of 32.)

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18. On Monday, Mukesh travelled x km at a constant speed of 90 km/h. On Tuesday, he travelled on the same route at a constant speed of 120 km/h. His trip on Tuesday took 16 minutes less than his trip on Monday. The value of x is

(A) 90 (B) 112 (C) 100 (D) 96 (E) 92

Source: 2018 Pascal Grade 9 #19

Primary Topics: Algebra and Equations

Secondary Topics: Rates

Answer: D

Solution:

We recall that $\text{time} = \dfrac{\text{distance}}{\text{speed}}$. Travelling x km at 90 km/h takes $\dfrac{x}{90}$ hours.
Travelling x km at 120 km/h takes $\dfrac{x}{120}$ hours.
We are told that the difference between these lengths of time is 16 minutes.
Since there are 60 minutes in an hour, then 16 minutes is equivalent to $\dfrac{16}{60}$ hours.
Since the time at 120 km/h is 16 minutes less than the time at 90 km/h, then $\dfrac{x}{90} - \dfrac{x}{120} = \dfrac{16}{60}$.
Combining the fractions on the left side using a common denominator of $360 = 4 \times 90 = 3 \times 120$, we obtain $\dfrac{x}{90} - \dfrac{x}{120} = \dfrac{4x}{360} - \dfrac{3x}{360} = \dfrac{x}{360}$.
Thus, $\dfrac{x}{360} = \dfrac{16}{60}$.
Since $360 = 6 \times 60$, then $\dfrac{16}{60} = \dfrac{16 \times 6}{360} = \dfrac{96}{360}$. Thus, $\dfrac{x}{360} = \dfrac{96}{360}$ which means that $x = 96$.

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19. Jeff and Ursula each run 30 km. Ursula runs at a constant speed of 10 km/h. Jeff also runs at a constant speed. If Jeff's time to complete the 30 km is 1 hour less than Ursula's time to complete the 30 km, at what speed does Jeff run?
(A) 6 km/h (B) 11 km/h (C) 12 km/h (D) 15 km/h (E) 22.5 km/h

Source: 2017 Pascal Grade 9 #11

Primary Topics: Geometry and Measurement | Number Sense

Secondary Topics: Rates

Answer: D

Solution:

When Ursula runs 30 km at 10 km/h, it takes her $\dfrac{30\text{ km}}{10\text{ km/h}} = 3\text{ h}$.
This means that Jeff completes the same distance in $3\text{ h} - 1\text{ h} = 2\text{ h}$.
Therefore, Jeff's constant speed is $\dfrac{30\text{ km}}{2\text{ h}} = 15\text{ km/h}$.

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20. Jiwei and Hari entered a race. Hari finished the race in $\frac{4}{5}$ of the time it took Jiwei to finish. The next time that they raced the same distance, Jiwei increased his average speed from the first race by x%, while Hari maintained the same average speed as in the first race. In this second race, Hari finished the race in the same amount of time that it took Jiwei to finish. The value of x is
(A) 20 (B) 25 (C) 35 (D) 40 (E) 50

Source: 2024 Cayley Grade 10 #20

Primary Topics: Algebra and Equations | Geometry and Measurement

Secondary Topics: Measurement | Expressions | Fractions/Ratios | Percentages

Answer: B

Solution:

Suppose that the length of the race was d m.
Suppose further that Jiwei finished the first race in t s.
Since Hari finished in $\dfrac{4}{5}$ of the time that Jiwei took, then Hari finished in $\dfrac{4}{5}t$ s.
Since speed equals distance divided by time, then Jiwei's average speed was $\dfrac{d}{t}$ m/s and Hari's average speed was $\dfrac{d}{4t/5} = \dfrac{5}{4} \cdot \dfrac{d}{t}$ m/s.
For Jiwei to finish in the same time as Hari, Jiwei must increase his average speed from $\dfrac{d}{t}$ m/s to $\dfrac{5}{4} \cdot \dfrac{d}{t}$ m/s.
This is an increase of one-quarter over the original speed, or an increase of 25%. Thus, $x = 25$.

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21. Integers m and n are each greater than 100. If $m + n = 300$, then $m : n$ could be equal to
(A) $9 : 1$ (B) $17 : 8$ (C) $5 : 3$ (D) $4 : 1$ (E) $3 : 2$

Source: 2005 Pascal Grade 9 #21

Primary Topics: Number Sense

Secondary Topics: Fractions/Ratios

Answer: E

Solution:

We try each of the five possibilities.
If $m : n = 9 : 1$, then we set $m = 9x$ and $n = x$, so $9x + x = 300$ or $10x = 300$ or $x = 30$, so~ $m = 9(30) = 270$ and $n = 30$.
If $m : n = 17 : 8$, then we set $m = 17x$ and $n = 8x$, so $17x + 8x = 300$ or $25x = 300$ or $x = 12$, so~ $m = 17(12) = 204$ and $n = 8(12) = 96$.
If $m : n = 5 : 3$, then we set $m = 5x$ and $n = 3x$, so $5x + 3x = 300$ or $8x = 300$ or $x = \frac{75}{2}$, so~ $m = 5(\frac{75}{2}) = \frac{375}{2}$ and $n = 3(\frac{75}{2}) = \frac{225}{2}$.
If $m : n = 4 : 1$, then we set $m = 4x$ and $n = x$, so $4x + x = 300$ or $5x = 300$ or $x = 60$, so~ $m = 4(60) = 240$ and $n = 60$.
If $m : n = 3 : 2$, then we set $m = 3x$ and $n = 2x$, so $3x + 2x = 300$ or $5x = 300$ or $x = 60$, so~ $m = 3(60) = 180$ and $n = 2(60) = 120$.
The only one of the possibilities for which m and n are integers, each greater than 100, \linebreak is $m : n = 3 : 2$.

22. Each time Kim pours water from a jug into a glass, exactly 10% of the water remaining in the jug is used. What is the minimum number of times that she must pour water into a glass so that less than half the water remains in the jug?
(A) 5 (B) 6 (C) 7 (D) 8 (E) 9

Source: 2009 Gauss Grade 7 #23

Primary Topics: Number Sense

Secondary Topics: Percentages | Fractions/Ratios

Answer: C

Solution:

Solution 1 We can suppose that the jug contains 1 litre of water at the start. The following table shows the quantity of water poured in each glass and the quantity of water remaining in each glass after each pouring, stopping when the quantity of water remaining is less than~0.5 L.

Number of glasses	Number of litres poured	Number of litres remaining
1	10% of $1 = 0.1$	$1 - 0.1 = 0.9$
2	10% of $0.9 = 0.09$	$0.9 - 0.09 = 0.81$
3	10% of $0.81 = 0.081$	$0.81 - 0.081 = 0.729$
4	10% of $0.729 = 0.0729$	$0.729 - 0.0729 = 0.6561$
5	10% of $0.6561 = 0.06561$	$0.6561 - 0.06561 = 0.59049$
6	10% of $0.59049 = 0.059049$	$0.59049 - 0.059049 = 0.531441$
7	10% of $0.531441 = 0.0531441$	$0.531441 - 0.0531441 = 0.4782969$

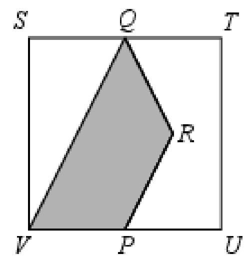
We can see from the table that the minimum number of glasses that Kim must pour so that less than half of the water remains in the jug is 7. Solution 2 Removing 10\% of the water from the jug is equivalent to leaving 90\% of the water in the jug. Thus, to find the total fraction remaining in the jug after a given pour, we multiply the previous total by 0.9. We make the following table, stopping when the fraction of water remaining in the glass is first less than 0.5 (one half).

Number of glasses poured	Fraction of water remaining
1	$0.9 \times 1 = 0.9$
2	$0.9 \times 0.9 = 0.81$
3	$0.9 \times 0.81 = 0.729$
4	$0.9 \times 0.729 = 0.6561$
5	$0.9 \times 0.6561 = 0.59049$
6	$0.9 \times 0.59049 = 0.531441$
7	$0.9 \times 0.531441 = 0.4782969$

We can see from the table that the minimum number of glasses that Kim must pour so that less than half of the water remains in the jug is 7.

23. In the diagram shown,
    - $STUV$ is a square,
    - Q and P are the midpoints of ST and UV,
    - $PR = QR$, and
    - VQ is parallel to PR.

    What is the ratio of the shaded area to the unshaded area?



(A) 2 : 3 (B) 3 : 5 (C) 1 : 1 (D) 7 : 9 (E) 5 : 7

Source: 2014 Gauss Grade 7 #24

Primary Topics: Geometry and Measurement

Secondary Topics: Area | Fractions/Ratios

Answer: B

Solution:

We begin by joining Q to P.
Since Q and P are the midpoints of ST and UV, then QP is parallel to both SV and TU and rectangles $SQPV$ and $QTUP$ are identical.
In rectangle $SQPV$, VQ is a diagonal.
Similarly, since PR is parallel to VQ then PR extended to T is a diagonal of rectangle $QTUP$, as shown in Figure 1.
In Figure 2, we label points A, B, C, D, E, and F, the midpoints of SQ, QT, TU, UP, PV, and VS, respectively.
We join A to E, B to D and F to C, with FC intersecting QP at the centre of the square O, as shown.
Since $PR = QR$ and R lies on diagonal PT, then both FC and BD pass through R. (That is, R is the centre of $QTUP$.)
The line segments AE, QP, BD, and FC divide square $STUV$ into 8 identical rectangles.
In one of these rectangles, $QBRO$, diagonal QR divides the rectangle into 2 equal areas.
That is, the area of $\triangle QOR$ is half of the area of rectangle $QBRO$.
Similarly, the area of $\triangle POR$ is half of the area of rectangle $PORD$.

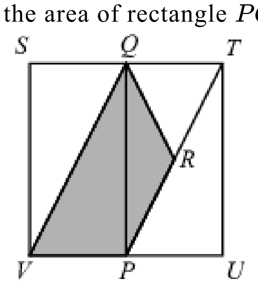


Figure 1

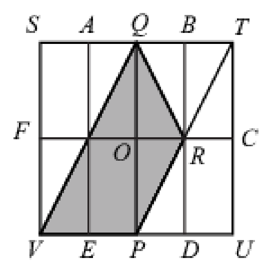
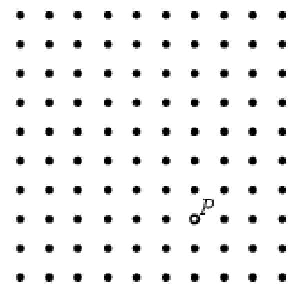


Figure 2

Rectangle $SQPV$ has area equal to 4 of the 8 identical rectangles.
Therefore, $\triangle QPV$ has area equal to 2 of the 8 identical rectangles (since diagonal VQ divides the area of $SQPV$ in half).
Thus the total shaded area, which is $\triangle QOR + \triangle POR + \triangle QPV$, is equivalent to the area of $\frac{1}{2} + \frac{1}{2} + 2$ or 3 of the identical rectangles.
Since square $STUV$ is divided into 8 of these identical rectangles, and the shaded area is equivalent to the area of 3 of these 8 rectangles, then the unshaded area occupies an area equal to that of the remaining $8 - 3$ or 5 rectangles.
Therefore, the ratio of the shaded area to the unshaded area is $3 : 5$.

24. A 10 by 10 grid is created using 100 points, as shown. Point P is given. One of the other 99 points is randomly chosen to be Q. What is the probability that the line segment PQ is vertical or horizontal?



(A) $\frac{2}{11}$ (B) $\frac{1}{5}$ (C) $\frac{1}{10}$ (D) $\frac{4}{25}$ (E) $\frac{5}{33}$

Source: 2016 Gauss Grade 7 #21

Primary Topics: Counting and Probability

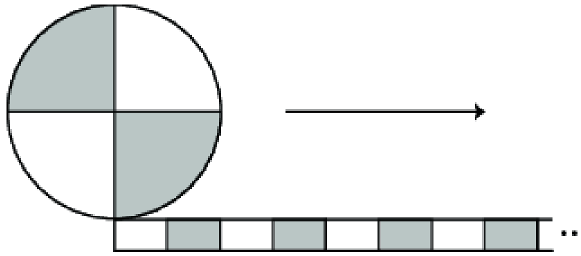
Secondary Topics: Probability | Fractions/Ratios

Answer: A

Solution:

Line segment PQ is vertical if Q is chosen from the points in the column in which P lies.
This column contains 9 points other than P which could be chosen to be Q so that PQ is vertical.
Line segment PQ is horizontal if Q is chosen from the points in the row in which P lies.
This row contains 9 points other than P which could be chosen to be Q so that PQ is horizontal.
Each of these 9 points is different from the 9 points in the column containing P.
Thus, there are $9 + 9 = 18$ points which may be chosen to be Q so that PQ is vertical or horizontal.
Since there are a total of 99 points to choose Q from, the probability that Q is chosen so that PQ is vertical or horizontal is $\frac{18}{99}$ or $\frac{2}{11}$.

25. A path of length 38 m consists of 19 unshaded stripes, each of length 1 m, alternating with 19 shaded stripes, each of length 1 m. A circular wheel of radius 2 m is divided into four quarters which are alternately shaded and unshaded. The wheel rolls at a constant speed along the path from the starting position shown.



The wheel makes exactly 3 complete revolutions. The percentage of time during which a shaded section of the wheel is touching a shaded part of the path is closest to

(A) 20% (B) 18% (C) 24% (D) 22% (E) 26%

Source: 2019 Cayley Grade 10 #23

Primary Topics: Geometry and Measurement

Secondary Topics: Circles | Percentages

Answer: E

Solution:

Since the wheel turns at a constant speed, then the percentage of time when a shaded part of the wheel touches a shaded part of the path will equal the percentange of the total length of the path where there is “shaded on shaded” contact.
Since the wheel has radius 2 m, then its circumference is $2\pi \times 2\text{ m}$ which equals $4\pi\text{ m}$.
Since the wheel is divided into four quarters, then the portion of the circumference taken by each quarter is $\pi\text{ m}$.
We call the left-hand end of the path 0 m.
As the wheel rotates once, the first shaded section of the wheel touches the path between 0 m and $\pi \approx 3.14$ m.
As the wheel continues to rotate, the second shaded section of the wheel touches the path between $2\pi \approx 6.28$ m and $3\pi \approx 9.42$ m.
While the wheel makes 3 complete rotations, a shaded quarter will be in contact with the path over 6 intervals (2 intervals per rotation).
The path is shaded for 1 m starting at each odd multiple of 1 m, and unshaded for 1 m starting at each even multiple of 1 m.
We make a chart of the sections where shaded quarters touch the path and the parts of these intervals that are shaded:

Beginning of quarter (m)	End of quarter (m)	Shaded parts of path (m)
0	$\pi \approx 3.14$	1 to 2; 3 to π
$2\pi \approx 6.28$	$3\pi \approx 9.42$	7 to 8; 9 to 3π
$4\pi \approx 12.57$	$5\pi \approx 15.71$	13 to 14; 15 to 5π
$6\pi \approx 18.85$	$7\pi \approx 21.99$	19 to 20; 21 to 7π
$8\pi \approx 25.13$	$9\pi \approx 28.27$	8π to 26; 27 to 28
$10\pi \approx 31.42$	$11\pi \approx 34.56$	10π to 32; 33 to 34

Therefore, the total length of “shaded on shaded”, in metres, is

$$1 + (\pi - 3) + 1 + (3\pi - 9) + 1 + (5\pi - 15) + 1 + (7\pi - 21) + (26 - 8\pi) + 1 + (32 - 10\pi) + 1$$

which equals $(16 - 2\pi)$ m.
The total length of the path along which the wheel rolls is $3 \times 4\pi$ m or 12π m.
This means that the required percentage of time equals $\dfrac{(16 - 2\pi)\text{ m}}{12\pi\text{ m}} \times 100\% \approx 25.8\%$.
Of the given choices, this is closest to 26%, or choice (E).